MACHAKOS UNIVERSITY

2018/2019

SMA 301: REAL ANALYSIS II - CAT I

Instructions: Answer ALL questions. [30 Marks] Date: **19/03/2019**

(1) If S is a finite set we sometimes write $\#S$ to denote the number of elements in S. Let $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$. For any $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ in $\mathbb{R}^n$ define

$$d(x, y) = \#\{i : x_i \neq y_i\}.$$

That is $d(x, y)$ are the number of components of x and y that are different componentwise. For instance if $x = (2, 3.1, \frac{1}{2})$ and $y = (2, \frac{1}{3}, \frac{1}{2})$ then $d(x, y) = 1$. Prove that d is a metric on $\mathbb{R}^3$. **[8 Marks]**

(2) Let X be a set and suppose that d_1 and d_2 are metrics on X. **[8 Marks]**

(i) Show that the function $d : X \times X \to \mathbb{R}$ defined by

$$d(x, y) = d_1(x, y) + d_2(x, y) \text{ for all } x, y \in X$$

is also a metric on X.

(ii) Let d_1 and d_∞ be the metrics defined on $\mathbb{R}^2$ by

$$d_1(x, y) = |x_1 - y_1| + |x_2 - y_2|$$
$$d_\infty(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$$

where $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are elements of $\mathbb{R}^2$. Calculate the distance

$$d(p, q)$$

where $p = (-1, -4), q = (6, 7) \in \mathbb{R}^2$ and where $d = d_1 + d_\infty$.

(3) Let d denote the usual metric on $\mathbb{R}$ and let A be the set $A = [4, 5] \cup (6, 7) \subset \mathbb{R}$. Is A and closed set in $(\mathbb{R}, d)$? Justify your answer. **[4 Marks]**

(4) Let (X, d) be a metric space and $A \subset X$ be a subset. **[5 Marks]**

(i) What do we mean by the term *limit points* of A?

(ii) Suppose $X = \mathbb{R}^2$ and that d is the Euclidean metric on $\mathbb{R}^2$. Let A be the set

$$A = \{x = (x_1, x_2) \in \mathbb{R}^2 : 3 \leq x_1 \leq 4, 2 \leq x_2 < 4\}.$$

What are the limit points of A?

(5) Let (X, d) be a metric space and $A, B \subset X$ be subsets of X. **[5 Marks]**

(a) What do we mean by the term *closure* of A?

(b) Show that $\overline{A} \cup \overline{B} = \overline{A \cup B}$.