UNIVERSITY OF NAIROBI
UNIVERSITY EXAMINATIONS 2020/2021
SECOND YEAR C.A.T FOR
THE DEGREES OF : BACHELOR OF SCIENCE (GENERAL), BACHELOR OF SCIENCE (CHEMISTRY),BACHELOR OF SCIENCE (GEOLOGY), BACHELOR OF SCIENCE (METEOROLOGY), BACHELOR OF SCIENCE (PETROLEUM GEOSCIENCE)
SMA 201: ADVANCED CALCULUS
MARCH 23, 2021 (4:40pM-6:40PM)

ATTEMPT QUESTION ONE AND ANY OTHER TWO QUESTIONS

Question 1 [30 marks]

(**a**) Find the **volume** of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the $xy-$plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

[4 marks]

(**b**) Find the **domain** of the function

$$f(x, y) = \frac{\sqrt{x + y + 1}}{x - 1}$$

and evaluate $f(3, 2)$.

[2 marks]

(**c**) If $f(x, y) = x^2 cosy + y^2 sinx$, verify that $f_{xy} = f_{yx}$.

[4 marks]

(**d**) If $f(x, y) = \frac{xy^2}{x^2+y^4}$, does $\lim\limits_{(x,y)\to(0,0)} f(x, y)$ exist?

[4 marks]

(**e**) Evaluate the iterated integral $\int_0^3 \int_1^2 x^2 y dy dx$

[4 marks]

(**f**) Find an approximate value for the integral $\int\int_R (x - 3y^2) dA$, where $R = \{(x, y) : 0 \leq x \leq 2, 1 \leq y \leq 2\}$, by computing the **double Riemann sum** with partition lines $x = 1$ and $y = \frac{3}{2}$ and taking (x^*_{ij}, y^*_{ij}) to be the center for each rectangle.

[4 marks]

(**g**)Find the **tangent plane** to the elliptic paraboloid $z = 2x^2 + y^2$ at the point $(1, 1, 3)$.

[4 marks]

(**h**) Evaluate $\int\int_D (x + 2y) dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$

[4 marks]

Question 2 [20 marks]

(**a**) Verify that

$$f(x,y) = ln(x^2 + y^2)$$

satisfies Laplace's equation

$$f_{xx} + f_{yy} = 0$$

[3 marks]

(**b**) If

$$U(x,y) = tan^{-1}(\frac{y}{x})$$

then verify that

$$U_{xx} + U_{yy} = 0$$

[3 marks]

(**c**) If

$$z = \frac{xy}{x-y}$$

show that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$$

[5 marks]

(**d**) Show that

$$f(x,t) = e^{(x-at)}$$

satisfies the wave equation

$$a^2 f_{xx} = f_{tt}$$

[3 marks]

(**e**) The profit function of a store rearing chicken is given by

$$P(p_1, p_2) = -3960 + 178p_1 + 274p_2 + 2p_1p_2 - 3p_1^2 - 2p_2^2$$

where p_1 is the retail price of a broiler, p_2 is the retail price (in dollars) of a toaster, and both p_1 and p_2 are non-negative. How should the retail store price its broilers and toasters to maximize profit?

[6 marks]

Question 3 [20 marks]

(**a**) A rectangular box without a lid is to be made from $12m^2$ of cardboard. Find the maximum volume of such a box using **Lagrange multipliers.**

[10 marks]

(**b**) Find the volume of the solid S that is bounded by the elliptic paraboloid $x^2+2y^2+z=16$, the planes $x=2$ and $y=2$, and the three coordinate planes.

[4 marks]

(**c**) Evaluate the iterated integral $\int_0^1 \int_{x^2}^{x^3} (x^2+y^2)dydx$

(**d**) Show that the function

$$U(x,y)=e^x siny$$

is a solution of Laplace's equation.

[3 marks]

Question 4 [20 marks]

(**a**) Define local minimum and local maximum values of a function $f(x,y)$ of two variables.

[2 marks]

(**b**) If $z=e^x siny$ where $x=st^2$ and $y=s^2t$, find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

[2 marks]

(**c**) Find and classify all the local extrema of the function

$$f(x,y)=x^4+y^4-4xy+1$$

[9 marks]

(**d**) Define gradient of a function of three variables x,y,z.

[2 marks]

(**e**) Find the gradient of the function

$$f(x,y,z)=xsin(yz)$$

[3 marks]

(**f**) If

$$z=f(x,y)=x^2+3xy-y^2$$

find the differential dz of the function.

[2 marks]

Question 5 [20 marks]

(**a**) Find the centre of mass for the thin plate bounded by the curves $g(x) = \frac{x}{2}$ and $f(x) = \sqrt{x}$, $0 \leq x \leq 1$ with the density function $\delta(x) = x^2$

[5 marks]

(**b**) Find the mass of the triangular lamina with vertices $(0, 0), (0, 3)$ and $(2, 3)$, given that the density at x, y is $\rho(x, y) = 2x + y$.

[3 marks]

(**c**) For what value of p is the integral

$$\int_1^{\infty} \frac{dx}{x^p}$$

convergent?

[4 marks]

(**d**) Evaluate

$$\int_0^3 \frac{dx}{x-1}$$

if possible.

[3 marks]

(**e**) The profit obtained by producing x units of product A and y units of product B is approximated by the model

$$p(x, y) = 8x + 10y - (0.001)(x^2 + xy + y^2) - 10000$$

Find the production level that produces a maximum profit.

[5 marks]