SCHOOL OF MATHEMATICS

SMA240: PROBABILITY AND STATISTICS I

TIME: 10.00am -12.00noon **CONTINUOUS ASSESSMENT TEST** **DATE: 8th April 2021**

INSTRUCTIONS:
- Answer QUESTION 1 or QUESTION 2 .
- Write your FULL NAME and ADMISSION NUMBER on each page of your solutions.
- Upload your solutions in ONE FILE. Solutions sent via email will not be marked.
- Use dark pen on size A4 pages for clarity.

QUESTION 1

(a) A random variable X has the following probability density function.

$$f(x) = ke^{-\lambda x}, \quad x > 0, \quad k \text{ is an unknown constant.}$$

(i) Express the constant k in terms of the parameter λ.

(ii) Determine the standard deviation of X using its moment generating function.

(b) The number of boys, X, in a family of eight children has a binomial distribution where the chances of a boy are equal to the chances of a girl being a member of the family. Determine the probability of the family having 2 boys or at least 4 girls.

(c) Records of a particular open-air kiosk indicate that over a certain period, profits, X, are normally distributed. Further information reveals that the chances of realizing a profit of at least £18.24 are 67% while the chances of realizing at most £27.84 are 97.5% .

(i) Determine the mean and standard deviation of the profits generated.

(ii) Determine $P\{|X - \mu| \leq 12\}$,where μ is the mean profit.

(d) Determine the number of times a fair die must be tossed so that the probability of the ratio of the number of fives realized to the number of tosses being between $^1/_{18}$ and $^5/_{18}$ is at least $^{15}/_{16}$.

QUESTION 2

(a) Consider the following joint probability density function of the random variables X and Y.

$$f(x, y) = kx^2ye^{-x}, \quad x > 0 ; \ 0 < y < 1,$$
$$= 0, \text{ otherwise.}$$

(i) Determine the value of the constant k. (ii) Compute $E(X^{-3}Y^4)^{-1}$

(ii) Determine the conditional mean of X^2 given Y. What can you say about the two random variables?

(b) Two independent random variables X_1 and X_2 are taken from population X which has the following probability density function.

$$\mathbf{f(x) = e^{-x}} \quad , x > 0$$
$$\mathbf{= 0}, \text{ otherwise}$$

Use change of variable technique to determine the probability density function of

$Z = X_1 X_2^{-1}$.

(c) Ten identically distributed random variables X_1, X_2, X_3, …….., X_{10} are taken from a normally distributed population X with mean 3 and variance 4. Write down an expression of the ten random variables which has (i) $N(0, 4)$ (ii) $F(4,5)$ (iii) $T(3)$
