MUEO

MOI UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR
(ACADEMIC AFFAIRS, RESEARCH & EXTENSION)

UNIVERSITY EXAMINATIONS

2020/2021 ACADEMIC YEAR

FOUTH YEAR FIRST SEMESTER EXAMINATION

FOR THE DEGREE OF

BACHELOR OF EDUCATION

COURSE CODE: MAT 404

COURSE TITLE: NUMERICAL METHODS

DATE: 28TH JANUARY, 2021 **TIME:** 2.00 P.M – 5.00 P.M

INSTRUCTION TO CANDIDATES

- SEE INSIDE

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UNIVERSITY EXAMINATIONS
2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

COURSE CODE: MAT 404
COURSE TITLE: NUMERICAL METHIDS
(MAIN EXAMINATION, JANUARY 2021)

INSTRUCTION TO CANDIDATES
Answer **ALL** questions from section A and any **THREE** from section B.
Illustrate your answers with suitable diagrams wherever necessary.
Duration of the examination: 3 hours

SECTION A (31 marks): Answer ALL questions

QUESTION ONE (15mks)

a) Use Laplace transform to solve the ODE $y'' - 4y' + 3y = e^{2t}$ subject to the conditions $y(0) = y'(0) = 0$ (5mks)

b) Show that $\Gamma(z + 1) = z!$ (5mks)

c) Show that the Fourier series coefficient a_n for the function $f(x) = x^3$ is zero. (5mks)

QUESTION TWO (16mks)

a) Use Taylor series of order 3 to solve the IVP $y'' - xy' + e^x y = 4$ sublect to the conditions $y(0) = 1,\ y'(0) = 4$ (6mks)

b) Using the definition ⍰$(\omega) = \frac{1}{\sqrt{2\pi}}\int_{\infty}^{\infty} f(x)\, e^{-i\omega x}dx$. Show that the Fourier transform of a Gaussian $f(x) = e^{-\frac{x^2}{2}}$ equals to a Gaussian $f(\omega) = e^{-\frac{\omega^2}{2}}$ (6mks)

c) Show that the Laplace transform of $f(x) = \sin \omega x$ equals to $\frac{\omega}{s^2+\omega^2}$ (4mks)

SECTION B **Attempt ANY THREE questions**
ALL Questions carry Equal Marks (13mks) each

QUESTION THREE

a) The population of chicken grows at a rate of r per season. Form a difference equation and find the closed form solution. If $x_0 = 300$, and $r = 0.05$, find the population after 6 seasons, assuming that no chicken was slaughtered. (5mks)

b) Determine the Fourier transform of $f(x) = \frac{1}{6}e^{-3|x|}$. (3mks)

c) Use power series to solve the ODE $y'' + 4y = 0$ subject to $y(0) = \frac{1}{3}$, $y'(0) = \frac{2}{5}$ and express your answer in trigonometric (sine and cosine) ratios. (5mks)

QUESTION FOUR

a) Form a Volterra Integral equation corresponding to the differential equation $y'' - 4y' + 4y = 0; y(0) = 3, y'(0) = 5.$ (5mks)

b) Find the Fourier series of the function $f(x) = \begin{cases} 0, & x < -2 \\ x^2, & -2 \le x \le 2 \\ 0, & x > 2 \end{cases}$ (5mks)

c) Obtain a system of two ordinary differential equation by separation of variables from the partial differential equation $u_t = ku_{xx}$. (3mks)

QUESTION FIVE

a) Find the inverse Laplace transform of $\frac{1}{s^2-3s+2}$ (4mks)

b) Compute the gamma function $\Gamma(\frac{1}{2})$ and show that $\Gamma\left(\frac{5}{2}\right) = \frac{3\sqrt{\pi}}{4}$ (5mks)

c) Formulate a mathematical mode using differential equations representing the reaction kinetics illustrated below (4mks)

$$S + E \underset{k_2}{\overset{k_1}{\rightleftarrows}} SE \overset{k_3}{\rightarrow} P + E$$

QUESTION SIX

a) Solve the Legendre equation $(1 - x^2)y'' - 2xy' + 2y = 0$ using power series method and express your answer in form of series up to $n = 5$. (6mks)

b) Use Fourier transform to solve the partial differential equation $u_t = \alpha^2 u_{xx}$, subject to $u(x, 0) = f(x)$ for $-\infty \le x \le \infty, t \ge 0$ (7mks)

QUESTION SEVEN

a) Find the Fourier transform of the function $f(x) = H(x)ke^{-2x}$ where the Heaviside function $H(t) = \begin{cases} 1, & x \ge 0 \\ 0, & \text{elsew□ere} \end{cases}$ (3mks)

b) Use Laplace operator □ to transform the Bessel function $ty'' + y' + ty = 0$ into a $s - domain$ and solve using the conditions $y(0) = y'(0) = 0$ (7mks)

c) Use the Beta function to evaluate $B(4, 3)$. (3mks)