



MUEO

# MOI UNIVERSITY

OFFICE OF THE DEPUTY VICE CHANCELLOR  
(ACADEMIC AFFAIRS, RESEARCH & EXTENSION)

## UNIVERSITY EXAMINATIONS 2020/2021 ACADEMIC YEAR FOURTH YEAR FIRST SEMESTER EXAMINATION

### FOR THE DEGREE OF BACHELOR OF EDUCATION

**COURSE CODE:** MAT 404

**COURSE TITLE:** NUMERICAL METHODS

**DATE:** 28<sup>TH</sup> JANUARY, 2021

**TIME:** 2.00 P.M - 5.00 P.M

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### INSTRUCTION TO CANDIDATES

- SEE INSIDE

THIS PAPER CONSISTS OF (3) PRINTED PAGES

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MOI UNIVERSITY

UNIVERSITY EXAMINATIONS  
2020/2021 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF  
BACHELOR OF EDUCATION

COURSE CODE: MAT 404

COURSE TITLE: NUMERICAL METHODS

(MAIN EXAMINATION, JANUARY 2021)

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**INSTRUCTION TO CANDIDATES**

Answer ALL questions from section A and any THREE from section B.

Illustrate your answers with suitable diagrams wherever necessary.

Duration of the examination: 3 hours  
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**SECTION A (31 marks): Answer ALL questions**

**QUESTION ONE (15mks)**

- a) Use Laplace transform to solve the ODE  $y'' - 4y' + 3y = e^{2t}$  subject to the conditions  $y(0) = y'(0) = 0$  (5mks)
- b) Show that  $\Gamma(z + 1) = z!$  (5mks)
- c) Show that the Fourier series coefficient  $a_n$  for the function  $f(x) = x^3$  is zero. (5mks)

**QUESTION TWO (16mks)**

- a) Use Taylor series of order 3 to solve the IVP  $y'' - xy' + e^x y = 4$  subject to the conditions  $y(0) = 1, y'(0) = 4$  (6mks)
- b) Using the definition  $\mathcal{F}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$ . Show that the Fourier transform of a Gaussian  $f(x) = e^{-\frac{x^2}{2}}$  equals to a Gaussian  $f(\omega) = e^{-\frac{\omega^2}{2}}$  (6mks)
- c) Show that the Laplace transform of  $f(x) = \sin \omega x$  equals to  $\frac{\omega}{s^2 + \omega^2}$  (4mks)

**SECTION B Attempt ANY THREE questions**

ALL Questions carry Equal Marks (13mks) each

**QUESTION THREE**

- a) The population of chicken grows at a rate of  $r$  per season. Form a difference equation and find the closed form solution. If  $x_0 = 300$ , and  $r = 0.05$ , find the population after 6 seasons, assuming that no chicken was slaughtered. (5mks)

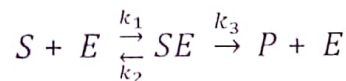
- b) Determine the Fourier transform of  $f(x) = \frac{1}{6}e^{-3|x|}$ . (3mks)
- c) Use power series to solve the ODE  $y'' + 4y = 0$  subject to  $y(0) = \frac{1}{3}$ ,  $y'(0) = \frac{2}{5}$  and express your answer in trigonometric (sine and cosine) ratios. (5mks)

#### QUESTION FOUR

- a) Form a Volterra Integral equation corresponding to the differential equation  $y'' - 4y' + 4y = 0$ ;  $y(0) = 3$ ,  $y'(0) = 5$ . (5mks)
- b) Find the Fourier series of the function  $f(x) = \begin{cases} 0, & x < -2 \\ x^2, & -2 \leq x \leq 2 \\ 0, & x > 2 \end{cases}$  (5mks)
- c) Obtain a system of two ordinary differential equation by separation of variables from the partial differential equation  $u_t = ku_{xx}$ . (3mks)

#### QUESTION FIVE

- a) Find the inverse Laplace transform of  $\frac{1}{s^2 - 3s + 2}$  (4mks)
- b) Compute the gamma function  $\Gamma(\frac{1}{2})$  and show that  $\Gamma(\frac{5}{2}) = \frac{3\sqrt{\pi}}{4}$  (5mks)
- c) Formulate a mathematical model using differential equations representing the reaction kinetics illustrated below (4mks)



#### QUESTION SIX

- a) Solve the Legendre equation  $(1 - x^2)y'' - 2xy' + 2y = 0$  using power series method and express your answer in form of series up to  $n = 5$ . (6mks)
- b) Use Fourier transform to solve the partial differential equation  $u_t = \alpha^2 u_{xx}$ , subject to  $u(x, 0) = f(x)$  for  $-\infty \leq x \leq \infty$ ,  $t \geq 0$  (7mks)

#### QUESTION SEVEN

- a) Find the Fourier transform of the function  $f(x) = H(x)ke^{-2x}$  where the Heaviside function  $H(t) = \begin{cases} 1, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$  (3mks)
- b) Use Laplace operator  $\mathcal{L}$  to transform the Bessel function  $ty'' + y' + ty = 0$  into a  $s$ -domain and solve using the conditions  $y(0) = y'(0) = 0$  (7mks)
- c) Use the Beta function to evaluate  $B(4, 3)$ . (3mks)

