UNIVERSITY OF EMBU

2019/2020 ACADEMIC YEAR

SUPPLEMENTARY/SPECIAL EXAMINATIONS

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (ECONOMICS &STATISTICS)

SMA 203: LINEAR ALGEBRA 1

DATE: OCTOBER 26, 2020 **TIME:** 8:30 AM – 10:30 AM

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE (30 MARKS)

a) For which values of b are the vectors $\{(2, -b), (2b + 6, 4b)\}$ linearly dependent (4 marks)

b) Differentiate between a consistent and an inconsistent system of equation (2 marks)

c) Do the vectors $\begin{bmatrix} x \\ y \\ x - 2y \end{bmatrix}$ form a subspace of $\mathbb{R}^3$? (5 marks)

d) Determine whether the set of exponential functions (e^{-3x}, e^{x}, e^{2x}) is linearly independent by calculating the WRONSKIAN (5 marks)

e) Prove that if $T: \boldsymbol{V} \to \boldsymbol{W}$ is a linear transformation, then the Kernel of T is a subspace of V (5 marks)

f) Determine whether the system of equation below is consistent or not

$$x + 2y + 3z = 14$$
$$x - 3y - 2z = -11$$

(5 marks)

g) Prove that if A is an invertible $n \times n$ matrix then for each b in $\mathbb{R}^n$ the equation $Ax = b$ has the unique solution $x = A^{-1}b$ (4 marks)

QUESTION TWO (20 MARKS)

a) For which value(s) of **k** does the following system have
   i) A unique solution
   ii) No solution
   iii) Infinitely many solutions

$$x + y + k = 2$$
$$3x + 4y + 2z = k$$
$$2x + 3y - z = 1$$

(8 marks)

b) Find the inverse of the matrix below by the adjoint matrix method

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{pmatrix}$$

Hence solve the following system of linear equations

$$2x_1 + x_2 + 3x_3 = 6$$
$$2x_1 + x_2 + x_3 = -12$$
$$4x_1 + 5x_2 + x_3 = 3$$

(12 marks)

QUESTION THREE (20 MARKS)

a) Solve the system of equations using LU decomposition method

$$2x_1 + x_2 - 2x_3 = 10$$
$$3x_1 + 2x_2 + 2x_3 = 1$$
$$5x_1 + 4x_2 + 3x_3 = 4$$

(8 marks)

b) Let T: $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by $T(x, y, z) = (2x - y, x + y + z, 2y - z)$ for each $\boldsymbol{x} = (x, y, z)$ in $\mathbb{R}^3$. Show that **T** is invertible and compute $\boldsymbol{T}^{-1}$ (7 marks)

c) Let $A = \begin{bmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is the equation $Ax = \boldsymbol{b}$ consistent for all values of $b_1\ b_2\ b_3$ (5 marks)

QUESTION FOUR (20 MARKS)

a) For the matrix A, let T: $\mathbb{R}^5 \to \mathbb{R}^4$ be define by $T(\boldsymbol{x}) = A(\boldsymbol{x})$ for each $\boldsymbol{x}$ in $\mathbb{R}^5$. Find the rank of T and a basis for image T (6 marks)

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 2 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 1 & 2 & 1 & -1 & 1 \end{bmatrix}$$

b) Use row reduction to determine the inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$,

hence solve the system of linear equations.

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4 \ .$$

(7 marks)

c) Let T: be $\mathbb{R}^2 \to \mathbb{R}^3$ be described by $\text{T}(x, y) = (x + y, x, 2x - y)$
Show that T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ (7 marks)

QUESTION FIVE (20 MARKS)

a) Is $S = \{(1,4,7), (2,5,8)(3,6,9)\}$ a basis for $\mathbb{R}^3$. (7 marks)

b) Let = $\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$, $\boldsymbol{u} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\boldsymbol{b} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$. $c = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and define a transformation.

T: $\mathbb{R}^2 \to \mathbb{R}^3$ by $T(\boldsymbol{x}) = A(\boldsymbol{x})$ so that $T(\boldsymbol{x}) = A(\boldsymbol{x}) = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix}$

i) Find $T(\boldsymbol{u})$ the image of $\boldsymbol{u}$ under the transformation T (2 marks)
ii) Find an $\boldsymbol{x}$ in $\mathbb{R}^2$ whose image under T is $\boldsymbol{b}$ (6 marks)
iii) Determine if $\boldsymbol{c}$ is in the range of the transformation **T** (5 marks)

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